UNIVERSITÄT ZU LÜBECK
INSTITUTE FOR ROBOTICS
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Humanoid Robotics
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Exercise sheet 0 - Matlab and V-rep

Please prepare the following exercises for the upcoming tutorial.

Task 1: Pendulum

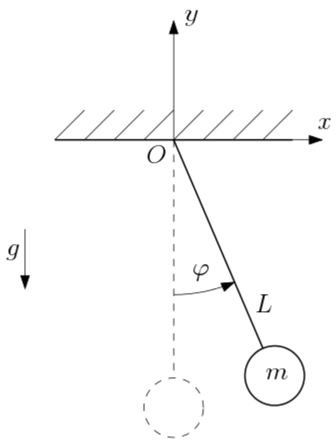

Figure 1 Pendulum

The pendulum, which is shown in Figure 1, shall be analyzed given the following specifications,

$$\begin{aligned} L &= 1.0\,\mathrm{m} \\ m &= 0.5\,\mathrm{kg} \\ g &= 9.81\,\mathrm{m/s^2} \\ \varphi(t=0) &= 0.1\,\mathrm{rad} \\ \dot{\varphi}(t=0) &= 0\,\mathrm{rad/s} \end{aligned} \tag{1}$$

Therefore, we start using the principle of momentum around the instantaneous centre of rotation, O.

$$\sum M^O = 0 = L\sin(\varphi)mg + mL^2\ddot{\varphi}. \tag{2}$$

This leads to the non-linear differential equation

$$\ddot{\varphi} + \frac{g}{L}\sin(\varphi) = \ddot{\varphi} + \omega^2\sin(\varphi) = 0; \qquad \omega = \sqrt{\frac{g}{L}}. \tag{3}$$

We are now able to linearize this equation under the assumption $\varphi << 1 \quad \forall t$. This leads to the linear differential equation

$$\ddot{\varphi} + \omega^2\varphi = 0. \tag{4}$$

The solution of this equation can be analytically calculated as

$$\varphi(t) = A\cos(\omega t) + B\sin(\omega t). \tag{5}$$

With the given initial conditions the solution of our linearized problem is

$$\varphi(t) = \varphi_0(t=0)\cos(\omega t). \tag{6}$$

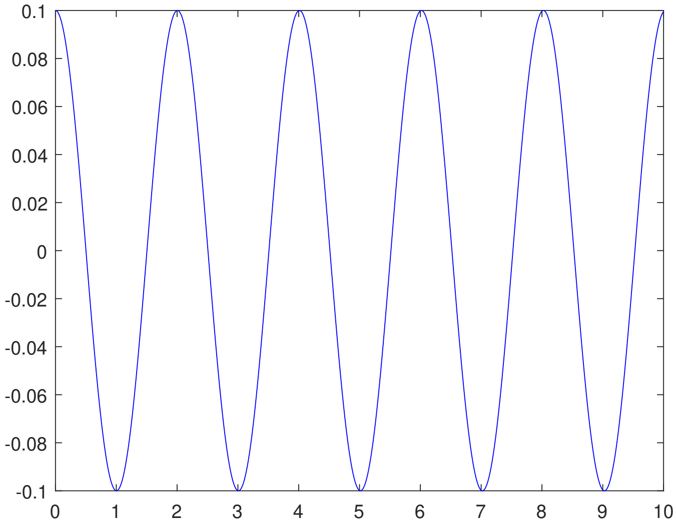

Figure 2 Solution linearized pendulum

The result is shown in Figure 2 for $t \in [0s, 10s]$. In order to estimate the movement of the pendulum using the non-linear differential equation, we can use the Matlab ODE (ordinary differential equation) solver, such as ode45. Using such a solver leads to solution shown in Figure 3. Both approaches lead to the same solution. If we are changing the initial conditions to $\varphi(t = 0) = 3.0\,\mathrm{rad}$, thus our assumption for the linearization does not hold anylonger, we see in Figure 4 that the linearized differential equation leads to unreliable results.

Task 2: Electric Circuit

The electric circuit, which is shown in Figure 5, shall be analyzed given the following specifications,

$$\begin{aligned}
V_0 &= 5.0\,\mathrm{V} \\
I_0 &= 2.0\,\mathrm{A} \\
R_1 &= 1.0\,\Omega \\
R_2 &= 1.0\,\Omega \\
R_3 &= 2.0\,\Omega \\
R_4 &= 2.0\,\Omega \\
R_5 &= 3.0\,\Omega \\
R_6 &= 3.0\,\Omega \\
R_7 &= 4.0\,\Omega \\
R_8 &= 4.0\,\Omega
\end{aligned} \tag{7}$$

Therefore we are using the node method. For this solution, we put the ground potential at the bottom and number the nodes as depicted in Figure 6. Using Kirchhoff's current law with the shown potentials leads to the following system of equations:

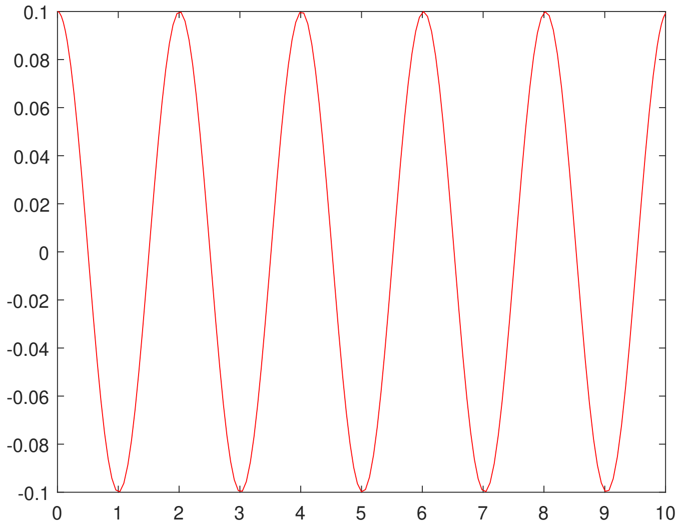

Figure 3 Solution non-linear Pendulum

$$\begin{bmatrix} G_1+G_3+G_4 & -G_3 & -G_4 & 0 \\ -G_3 & G_2+G_3+G_5 & 0 & -G_5 \\ -G_4 & 0 & G_4+G_6+G_7 & -G_6 \\ 0 & -G_5 & -G_6 & G_5+G_6+G_8 \end{bmatrix} \begin{bmatrix} e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} = \begin{bmatrix} V_0 G_1 \\ V_0 G_2 \\ 0 \\ I_0 \end{bmatrix}. \tag{8}$$

This system can be solved using Matlab which leads to

$$\begin{bmatrix} e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} = \begin{bmatrix} 4.7050 \\ 4.9903 \\ 3.8297 \\ 5.3891 \end{bmatrix}. \tag{9}$$

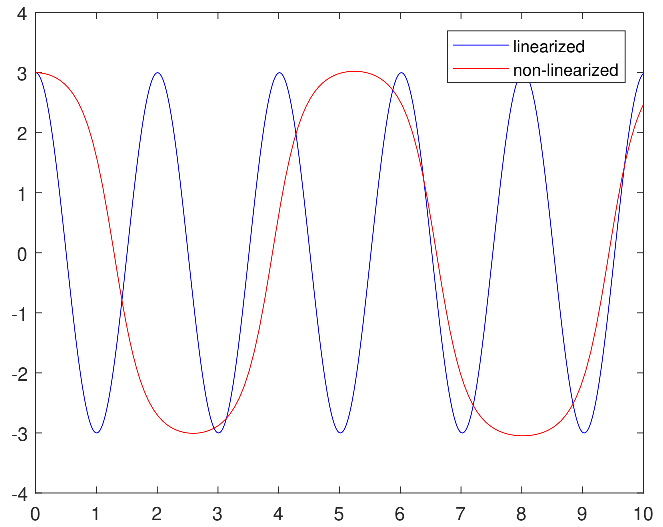

Figure 4 Comparison between linearized and non-linearized approach

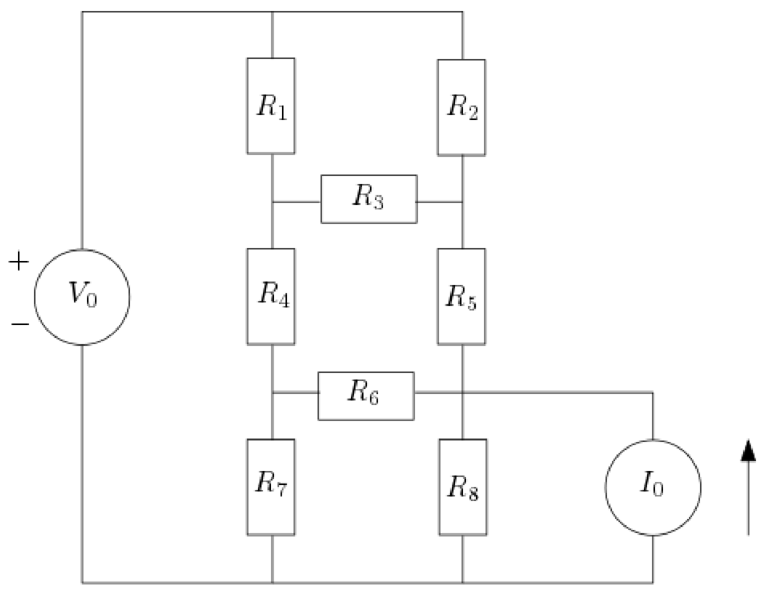

Figure 5 Electric Circuit

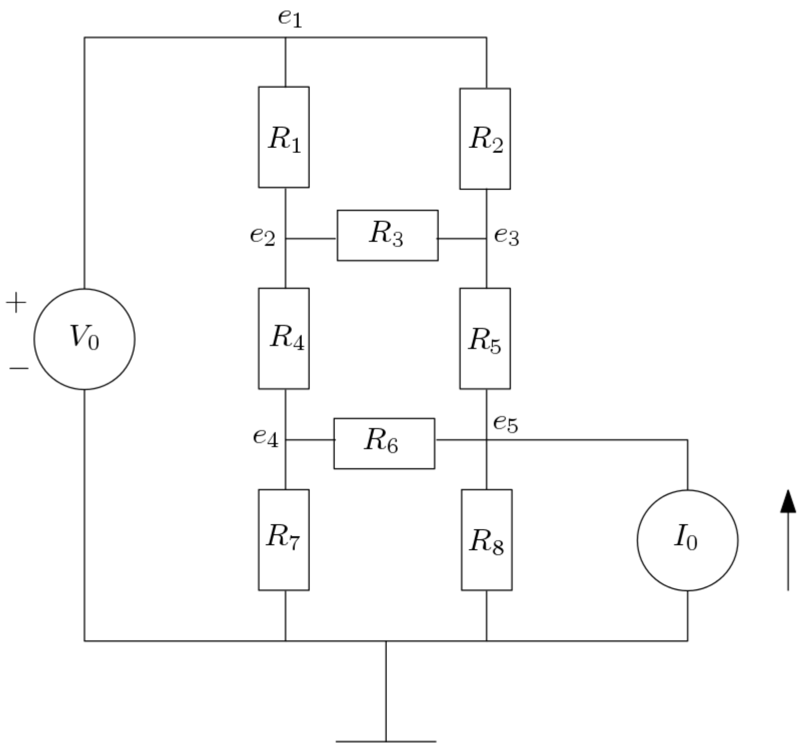

Figure 6 Electric Circuit with Potentials